

# Students Projects

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## Learning Symmetries by MetaLearning

### Question

How to parametrize the symmetries independently of the dimension of input vectors?

### Suggested Approach

To characterize the symmetry group by equalities in matrix space. For example orthogonal matrices  $O$  is defined by equality  $O \cdot O^T = E$ , where  $E$  is identical matrix. This definition does not depend on the dimension of the vector space.

### Related Work

- Zhou, A., Knowles, T., Finn, C. (2020). Meta-learning symmetries by reparameterization.
- Hyland, Stephanie, and Gunnar Rätsch. "Learning unitary operators with help from  $u(n)$ ." Proceedings of the AAAI Conference on Artificial Intelligence. Vol. 31. No. 1. 2017.

### Context

Convolutional neural networks (CNNs) have made great success in computer vision problems. One of the reasons CNNs perform well is because they are equivalent to translations. In recent years, several neural networks have been developed that are equivalent to a specific symmetry group, such as rotation, translation, etc. But it is difficult to find the symmetry group directly from the data and construct an equivariant NN using this knowledge. In this project, we propose to study a recent paper that simultaneously learns the symmetry group and equivariant fibers in NN. One of the drawbacks of the approach in the paper is a big amount of trainable parameters of the symmetry matrices. We propose to study the question of how many parameters are needed and formulate this problem mathematically.

# Invariant and Equivariant NN

## Question

Can we create a neural network invariant to the action of some group  $G$

## Suggested Approach

For many groups it is known how to characterize functions that are invariant to the action of the group. Reynolds operator projects the function to the space of invariant functions. For example for a permutation group  $S_n$  acting naturally on the set of  $n$  elements, Reynold operator look the following:

$$R : f(x) \rightarrow f\left(\sum_{\sigma \in S_n} \sigma(x)\right) \quad (1)$$

We propose to use the available description of Reynolds operator to create invariant and equivariant neural network.

## Related Work

- Zaheer, Manzil, et al. "Deep sets." arXiv preprint arXiv:1703.06114 (2017).
- Sturmfels, Bernd. Algorithms in invariant theory. Springer Science(2008)

## Context

In many applications the amount of the data is not enough for training robust neural network. But it is known that the classifier should be invariant for some transformation of the input. To overcome this issue, there were proposed architecture of Neural Networks that are invariant to the specif transformations of the input. The neural network that are invariant towards the permutation of the input are called Deep Sets. We propose to derive the same theory for more general groups using invariant theory.